No.12 略解

問 1

(1)

$$\begin{aligned}\mathcal{L}\left[\cos(t-2)u(t-2)\right] &= e^{-2s}\mathcal{L}\left[(\cos t)u(t)\right] \\ &= e^{-2s}\frac{s}{s^2+1}\end{aligned}$$

(2)

$$\begin{aligned}\mathcal{L}\left[(t+2)u(t-2)\right] &= \mathcal{L}\left[((t-2)+4)u(t-2)\right] \\ &= e^{-2s}\mathcal{L}\left[(t+4)\right] \\ &= e^{-2s}\left(\frac{1}{s^2}+\frac{4}{s}\right) \\ &= e^{-2s}\frac{4s+1}{s^2}\end{aligned}$$

(3)

$$\begin{aligned}\mathcal{L}\left[\sin t\, u(t-\frac{\pi}{2})\right] &= e^{-\frac{\pi}{2}s}\mathcal{L}\left[\sin(t+\frac{\pi}{2})\right] \\ &= e^{-\frac{\pi}{2}s}\mathcal{L}\left[\cos t\right] \\ &= e^{-\frac{\pi}{2}s}\frac{s}{s^2+1}\end{aligned}$$

(4)

$$\begin{aligned}\mathcal{L}\left[(t^2+1)u(t-2)\right] &= e^{-2s}\mathcal{L}\left[\{(t+2)^2+1\}u(t)\right] \\ &= e^{-2s}\mathcal{L}\left[(t^2+4t+5)u(t)\right] \\ &= e^{-2s}\left(\frac{2}{s^3}+\frac{4}{s^2}+\frac{5}{s}\right) \\ &= e^{-2s}\left(\frac{5s^2+4s+2}{s^3}\right)\end{aligned}$$

問 2

$f(t)$ はステップ関数 $u(t)$ を使って

$$f(t) = u(t)-3u(t-a)+2u(t-2a)$$

と表せるので

$$\begin{aligned}F(s) &= \mathcal{L}[f(t)] \\ &= \frac{1}{s}\left(1-3\mathrm{e}^{-as}+2\mathrm{e}^{-2as}\right) \\ &= \frac{1}{s}(1-\mathrm{e}^{-as})(1-2\mathrm{e}^{-as})\end{aligned}$$

問 3

$f(t)$ は以下のように表せる．

$$\begin{aligned}f(t) &= \sin t\cdot u(t)+2\sin(t-\pi)\cdot u(t-\pi) \\ &\quad +\sin(t-2\pi)\cdot u(t-2\pi)\end{aligned}$$

したがって，$F(s)$ は

$$\begin{aligned}F(s) &= \mathcal{L}[\sin t]+2\mathrm{e}^{-\pi s}\mathcal{L}[\sin t]+\mathrm{e}^{-2\pi s}\mathcal{L}[\sin t] \\ &= (1+2\mathrm{e}^{-\pi s}+\mathrm{e}^{-2\pi s})\mathcal{L}[\sin t] \\ &= \frac{(1+e^{-\pi s})^2}{s^2+1}.\end{aligned}$$

グラフは以下の通り．