

## No.10 略解

### 問 1

(1)

$$\begin{aligned}\mathcal{L}[\cos(t-2)u(t-2)] &= e^{-2s}\mathcal{L}[(\cos t)u(t)] \\ &= e^{-2s}\frac{s}{s^2+1}\end{aligned}$$

(2)

$$\begin{aligned}\mathcal{L}[(t+2)u(t-2)] &= \mathcal{L}[(t-2+4)u(t-2)] \\ &= e^{-2s}\mathcal{L}[(t+4)] \\ &= e^{-2s}\left(\frac{1}{s^2} + \frac{4}{s}\right) \\ &= e^{-2s}\frac{4s+1}{s^2}\end{aligned}$$

(3)

$$\begin{aligned}\mathcal{L}\left[\sin t u\left(t - \frac{\pi}{2}\right)\right] &= e^{-\frac{\pi}{2}s}\mathcal{L}\left[\sin\left(t + \frac{\pi}{2}\right)\right] \\ &= e^{-\frac{\pi}{2}s}\mathcal{L}[\cos t] \\ &= e^{-\frac{\pi}{2}s}\frac{s}{s^2+1}\end{aligned}$$

(4)

$$\begin{aligned}\mathcal{L}[(t^2+1)u(t-2)] &= e^{-2s}\mathcal{L}[\{(t+2)^2+1\}u(t)] \\ &= e^{-2s}\mathcal{L}[(t^2+4t+5)u(t)] \\ &= e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{5}{s}\right) \\ &= e^{-2s}\left(\frac{5s^2+4s+2}{s^3}\right)\end{aligned}$$

### 問 2

$f(t)$  はステップ関数  $u(t)$  を使って

$$f(t) = u(t) - 3u(t-a) + 2u(t-2a)$$

と表せるので

$$\begin{aligned}F(s) &= \mathcal{L}[f(t)] \\ &= \frac{1}{s}(1 - 3e^{-as} + 2e^{-2as}) \\ &= \frac{1}{s}(1 - e^{-as})(1 - 2e^{-as})\end{aligned}$$

### 問 3

$f(t)$  は以下のように表せる .

$$\begin{aligned}f(t) &= \sin t \cdot u(t) + 2\sin(t-\pi) \cdot u(t-\pi) \\ &\quad + \sin(t-2\pi) \cdot u(t-2\pi)\end{aligned}$$

したがって,  $F(s)$  は

$$\begin{aligned}F(s) &= \mathcal{L}[\sin t] + 2e^{-\pi s}\mathcal{L}[\sin t] + e^{-2\pi s}\mathcal{L}[\sin t] \\ &= (1 + 2e^{-\pi s} + e^{-2\pi s})\mathcal{L}[\sin t] \\ &= \frac{(1 + e^{-\pi s})^2}{s^2 + 1}.\end{aligned}$$

グラフは以下の通り

